Interaction of Radiation with Turbulence: Application to a Combustion System

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Turbulence/radiation interaction is examined in order to provide better fundamental understanding of temporal aspects of radiative transfer in combustion systems. Two aspects of radiative transfer in a turbulent medium are considered in this paper. In the first, transfer of radiation along a path with turbulent concentration and temperature fluctuations is calculated for the time-mean irradiance at a combustion chamber wall due to random concentration of absorbing species and emission with Gaussian probability density functions. In the second, turbulence/radiation interaction and the effect of radiation transfer on the fully coupled structure and mean properties are assessed for an industrial natural gas-fired furnace. The results of calculations based on the approximate formulation utilized here show that the effects of turbulence/radiation interaction on combustion and flow properties is relatively small for a preheated methane-air mixture. The interaction is greater when the oxidant is cold and the flame is relatively long.

Nomenclature

A,B	= wavelength boundaries of spectral groups
C	= concentration
E_b	= blackbody emitted flux
F_λ	= radiative flux vector, $\int_{\Omega=4\pi} I_\lambda \vec{s} \mathrm{d}\Omega$
G_λ	= irradiance, $\int_{\Omega=4\pi} I_\lambda \mathrm{d}\Omega$
H	= height of furnace, see Figs. 1 and 2
I	= intensity of radiation
k	= thermal conductivity or turbulent kinetic energy
L	= length of furnace
M	= mass fraction
Q	= heat input rate
q	= heat flux
R	= turbulent correlation
r	= Favre weighting factor
S	= source
s	= coordinate along beam path
T	= temperature
u	= velocity
U_∞	= overall heat loss coefficient
w_{ij}	= weighting factor of spectral group ij diffusion coefficient
Γ	= diffusion coefficient
ϵ	= emissivity or dissipation of turbulent kinetic energy
η_λ	= volumetric emission coefficient, $\kappa_\lambda I_{b\lambda}(T)$
κ	= absorption coefficient
Λ	= turbulent microscale
μ	= dynamic viscosity
ϕ	= scalar quantity
ρ	= density

Subscripts

eff	= effective
fu	= fuel
in	= incoming
n	= normal to the wall
w	= wall
λ	= wavelength or per unit wavelength interval
1	= fresh mixture
∞	= ambient

I. Introduction

INTERACTION of convection and radiation has been recognized for some time,[1] but that radiative transfer can influence turbulence and vice versa has been identified in a number of more recent applications.[2-8] The first attempt of combined analysis of the equations of the mean-square fluctuations of the velocity and temperature fields with the radiation field is due to Townsend.[9] In geophysical applications, for example, it has been determined that radiative transfer can influence turbulence in the planetary atmospheres by shaping the temperature stratification and through the radiative damping affect the turbulent temperature fluctuations.[2,3] Analysis has been performed to determine the importance in turbulent fluctuations of thermal radiation from flames.[4] The results have implications to exhaust plume diagnostics, signature predictions, as well as diagnostic measurements in a wide class of turbulent flames. Other applications in which radiation/turbulence interaction may affect flow include industrial furnaces,[5] gas combustors,[6] flames,[7] and fires.[8] Since concentrations and temperature of the radiating species fluctuate, prediction of radiative transfer of necessity requires stimulation of the problem by representing the radiating volume of the system through its time-averaged properties. This procedure involves an unknown degree of approximation.

Contemporary turbulent convection/radiation interaction formulations[10,11] have neglected the effect of turbulence and predicted radiative transfer on the basis of mean temperature. Modeling of radiative transfer in combustion chambers and furnaces has also ignored turbulence/radiation interation.[12,13] However, the findings based on treating the combined effects of fluctuating absorption coefficient and temperature suggest that fluctuations can increase radiative fluxes up to 2-3 times higher than estimates based on mean properties.[14] An up-to-date discussion of turbulence/radiation interaction in flames is available[8] and need not be repeated here. It is sufficient to conclude that the interaction and coupled effects are more important for luminous than for nonluminous flames. Little is known concerning temporal aspects of radiative transfer in turbulent flames as these effects have not been studied exten-

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sively; therefore, additional study of turbulence/radiation interaction is needed.

An up-to-date review of predicting radiative transfer in multidimensional geometries is available[15] and need not be repeated here. However, it should be mentioned that differential approximations or multiflux methods offer computational efficiencies in comparison to methods such as Monte Carlo, zonal, hybrid, and others. These approximations are also compatible with numerical algorithms for solving the transport equations.

Turbulence/radiation interactions and coupled effects of radiation and flame structure have not been studied extensively and little is known concerning radiation transfer in turbulent flames. Simulation of radiation-turbulence interaction in simple turbulent flows has been recommended[8] to provide a better fundamental understanding of the process. To this end, this paper examines turbulence/radiation interactions in a radiating, turbulent jet and in an industrial combustion furnace to determine if the interaction causes radiative transfer to be significantly different from that estimated based on mean properties and temperatures.

II. Radiative Transfer in Turbulent Flows

For an absorbing-emitting but nonscattering gas the radiative transfer equation (RTE), expressing an instantaneous radiant energy balance on a pencil of radiation propagating in direction $\vec{s}$ and confined to a solid angle dΩ and specral interval λ and $\lambda + d\lambda$ is given by

$$(\vec{s}\cdot\nabla)I_\lambda = -\kappa_\lambda[I_\lambda - I_{b\lambda}(T)] \tag{1}$$

Integration of Eq. (1) over all directions yields an instantaneous spectral radiant energy balance on the gas,

$$\nabla\cdot\vec{F}_\lambda = -\kappa_\lambda[G_\lambda - 4E_{b\lambda}(T)] \tag{2}$$

The right-hand side of Eq. (2) represents the net rate of gain (loss) of spectral energy per unit volume. It is either the left- or right-hand side of Eq. (2), integrated over the entire spectrum that appears in the thermal energy balance for the temperature (enthalpy) distribution in the fluid.

The RTE, Eq. (1), can be integrated along the direction of propagation $\vec{s}$ to yield an expression for the spectral intensity of radiation (radiance) to yield

$$I_\lambda(s,t) = I_{0\lambda}\exp\left[-\int_0^s \kappa_\lambda(s',t)\mathrm{d}s'\right] + \int_0^s \kappa_\lambda(s',t)I_{b\lambda}[T(s',t)]\exp\left[\int_s^s \kappa_\lambda(s'',t)\mathrm{d}s\right]\mathrm{d}s' \tag{3}$$

where $I_{0\lambda}$ denotes the intensity of radiation incident on the boundary (at $s=0$) of the medium. The first term on the right-hand side represents the contribution to intensity due to boundary radiation and the second due to emission and reabsorption of radiation by the medium.

Depending on the particular application, the objective is to evaluate a time-average of Eqs. (1), (2), or (3), including the contributions from turbulent fluctuations of temperature and spectral absorption coefficient. If the absorption coefficient can be expressed as

$$\kappa_\lambda(s,t) = \sum_i k_{\lambda i}C_i(s,t) \tag{4}$$

the turbulent fluctuations in the absorption coefficient can be related to those in the concentrations C_i's of the radiating species and the temperature T. The precise evaluation of the time-average would utilize the joint probability density function $p(T,C_i,s)$ of the temperature and species concentrations for all points s along the line of sight $\vec{s}$ in Eq. (3). Unfortunately, that information is not available. Those properties of the flowfield that are available are the mean temperature $\bar{T}$, species concentrations $\bar{C}_i$, and the second order correlations, $\overline{T'^2}$, $\overline{C_i'T'}$, and $\overline{C_i'^2}$. The approach which has been taken[4,5,16] is to expand each variable into its mean and fluctuating part, and to substitute directly into Eq. (3), keeping terms up to second-order in the fluctuations.

The fluctuations of the Planck function $I_{b\lambda}(T)$ and the spectral absorption coefficient $\kappa_\lambda(T)$ can be given in terms of the temperature and species fluctuations by means of Taylor series expansions about the values evaluated at the mean properties. Evaluation of the instantaneous intensity of radiation in terms of the mean and fluctuating properties, and the time-averaging, is a straightforward procedure, but it is tedious. Results can be found elsewhere.[4]

To illustrate the nature of the problem we restrict ourselves to a single radiating species and assume that the spectral emission rate $\eta_\lambda = \kappa_\lambda I_{b\lambda}(T)$ and the concentration C can be expressed as random, stationary fields with Gaussian probability distributions.[5] Applying Reynolds' averaging techniques to Eq. (3) but omitting the details, one can obtain

$$\begin{aligned}\bar{I}_\lambda(s) = {} & I_{0\lambda}\exp\left[-k_\lambda\int_0^s C(s')\mathrm{d}s'\right]\overline{\exp\left[-k_\lambda\int_0^s C'(s')\mathrm{d}s'\right]} \\ & + \int_0^s \bar{\eta}_\lambda\exp\left[-k_\lambda\int_{s'}^s \bar{C}\mathrm{d}s''\right]\left\{\overline{\exp\left[-k_\lambda\int_{s'}^s C'\mathrm{d}s''\right]}\right. \\ & \left. + \left(\frac{1}{\bar{\eta}_\lambda}\right)\overline{\eta_\lambda'\exp\left[-k_\lambda\int_{s'}^s C'\mathrm{d}s''\right]}\right\}\mathrm{d}s'\end{aligned} \tag{5}$$

Equation (5) can be written in a more useful form in terms of the two-point correlation coefficients.[5] The representation of the random concentration and temperature by Gaussian variables is convenient but it must be noted that they encompass unrealistic negative values of the variable whose probability must be kept small in proportion. The approach of calculating the time-mean radiation field using the time-averaged spectral radiance is discussed in the next section. Comparison of Eqs. (3) and (5) reveals that turbulence (i.e., time-averaging) has greatly increased the computational effort of an already difficult problem.

An alternative to time-averaging the spectral radiance would be to time-average the RTE, Eq. (1), and the radiant energy equation, Eq. (2), at the start. Time-averaging of Eq. (1) results in

$$(\vec{s}\cdot\nabla)\bar{I}_\lambda = -\overline{\kappa_\lambda I_\lambda} + \bar{\eta}_\lambda \tag{6}$$

The difficulty with this equation is the coupling $\overline{\kappa_\lambda I_\lambda}$ since instantaneous I_λ is expressed in terms of an integration along the path as indicated in Eq. (3). To simplify the absorption coefficient-radiance correlation $\overline{\kappa_\lambda I_\lambda}$, physically plausible approximations will have to be made. This will be discussed in Sec. IV.

III. Radiative Transfer in a Turbulent Jet: An Example

To estimate the turbulence-radiation interaction and obtain a feel for its magnitude, we consider a simple example shown in Fig. 1. A planar turbulent-jet of combustion gases is assumed to be confined between two isothermal parallel plates. The random concentration and temperature are represented in terms of Gaussian variables. In terms of the two-point concentration correlation coefficient,

$$R_{CC} = \frac{\overline{C'(y)C'(y-\xi)}}{\sqrt{\overline{C'^2}(y)}\sqrt{\overline{C'^2}(y-\xi)}} \tag{7}$$

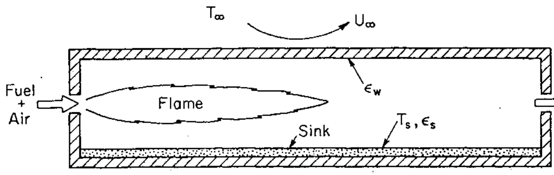

Fig. 1 Schematic diagram of a turbulent planar jet enclosed by two symmetrically located walls.

and similarly the two-point emission-concentation correlation coefficient,

$$R_{\eta C}=\frac{\overline{\eta'(y)C'(y-\xi)}}{\sqrt{\overline{\eta'^2}(y)}\sqrt{\overline{C'^2}(y-\xi)}} \tag{8}$$

The approximation that the emission-concentration correlation function is of the same form as the two-point concentration correlation would not be appropriate for a sooting flame, for example, where the absorption coefficient and temperature are likely to interact in some complex fashion.

Equation (5) can be specialized, and the spectral intensity at the upper plate $(y=H)$ in the direction normal to the plate can be expressed as

$$\begin{aligned}\bar{I}_\lambda(H)&=I_{0\lambda}\exp\left[-\int_0^H k_\lambda\bar{C}(y)\mathrm{d}y\right]\exp\left[\frac{1}{2}k_\lambda^2D(0,H)\right]\\&+\int_0^H\bar{\eta}\exp\left[-\int_{y'=y}^H k_\lambda\bar{C}(y')\mathrm{d}y'\right]\left\{\exp\left[\frac{1}{2}k_\lambda^2D(y,H)\right]\right.\\&\left.-\left[\frac{k_\lambda}{\bar{\eta}_\lambda}\right]\sqrt{\overline{\eta'^2}(y')}\int_{\xi=0}^{H-y}R_{\eta C}\sqrt{\overline{C'^2}(y-\xi)}\mathrm{d}\xi\right\}\mathrm{d}y\end{aligned} \tag{9}$$

where the function

$$D(y,H)=\int_{z_1=y}^H\int_{z_2=y}^H C'(z_1)C'(z_2)\mathrm{d}z_1\mathrm{d}z_2 \tag{10}$$

may be written as

$$D(y,H)=\int_{\xi=0}^{H-y-s}\int_{s=0}^{H-y}R_{CC}\sqrt{\overline{C'^2}(s+y)}\sqrt{\overline{C'^2}(s+y+\xi)}\mathrm{d}s\mathrm{d}\xi \tag{11}$$

As can be seen from Eq. (9) the turbulent fluctuations in concentration increase the transmittance by a factor of $\exp[1/2k_\lambda^2(D(0,H))]$. The fluctuations render the gas more transparent to radiation that would be predicted from its time-averaged properties. Note also that the second term containing mean values of $\bar{\eta}_\lambda$ and $\bar{C}$ is multiplied by a term that involves fluctuations of emission coefficient and concentration of the radiating species.

In order to clarify further the nature of increased transmittance due to concentration fluctuations of factor $\exp[1/2k_\lambda^2D(0,H)]$, we assumed a statistically uniform radiating medium which implies that

$$\sqrt{\overline{C'^2}(y)}=\sqrt{\overline{C'^2}(y-\xi)}\text{ and }R_{CC}(y)=R_{CC}(y-\xi)$$

and employ the correlation coefficient based on measurements made in a hot axisymmetric jet.[17] Use of the emission-concentration correlation coefficient $R_{\eta C}$ reported in the literature[17] implies that the opacity of the radiation participating gas based on the eddy size is sufficiently large; therefore, there is a correlation between absorption (concentration) and emission.

Table 1 Effect of flunctuation intensity and of optical depth on parameter $\exp[½k_\lambda^2D(0,H)]$: $k_\lambda\bar{C}=1\ \mathrm{m}^{-1}$ (independent of position)

Local intensity	Optical depth, $k_\lambda\bar{C}H$ 1.0	2.0	4.0
	Relative turbulence scale, $\Lambda/H=0.1$		
0.15	1.002	1.010	1.039
0.30	1.010	1.039	1.165
0.45	1.022	1.090	1.410
	Relative turbulence scale, $\Lambda/H=0.5$		
0.15	1.009	1.038	1.162
0.30	1.038	1.162	1.822
0.45	1.088	1.401	3.856

Table 2 Effect of turbulent intensity and correlation on time-averaged intensity of emission by a turbulent gas, $\bar{I}_\lambda(H)/I_\lambda(H)$, $\Lambda=0.15$

Correlation	Optical depth, $k_\lambda\bar{C}H$ 1	2	4
Local concentration and emission intensity, $\sqrt{\overline{C'^2}}/\bar{C}=\sqrt{\overline{\eta'^2}}/\bar{\eta}=0.3$			
+	0.9916	0.9911	0.9798
−	1.0199	1.0450	1.0720
Local concentration and emission intensity, $\sqrt{\overline{C'^2}}/\bar{C}=\sqrt{\overline{\eta'^2}}/\bar{\eta}=0.45$			
+	0.9810	0.9767	1.0034
−	1.0447	1.0967	1.2143

Sample calculations have been performed for the case when there is no emission from the gas $(\bar{\eta}_\lambda=0)$ and the results are summarized in Table 1. The double integrations to evaluate $D(0,H)$ were performed numerically. From Eq. (9) and the results of the table it is apparent that the transmittance, $\bar{I}_\lambda(H)/I_{0\lambda}$, of the gas can be greatly increased by the presence of turbulent concentration fluctuations. The effect increases with increasing intensity levels, opacity, and scale of turbulence. It may be noted, however, that for low intensity levels and opacities the effect is quite small. The trends in transmittance predicted for a turbulent, statistically uniform path of gas agree with the earlier results of Tan and Foster.[5]

We now consider the effect of concentration and emission fluctuations on the radiance in a turbulent gas. For a lack of better data, the concentration correlation was taken from the literature.[17] Even though this function was obtained for a jet of hot gases, it exhibits the same basic properties what would be expected for confined flow. As the first approximation, the emission-concentration correlation function $R_{\eta C}$ was assumed to be of the same form as the two-point concentration correlation, except that we considered two cases of the correlation function. For simplicity, we assume that both positive and negative correlations are modeled by the same function. The numerical results are summarized in Table 2.

It is apparent from the numerical results of Table 2 that the radiance in a turbulent gas depends on whether the concentration and emission fluctuations are positively or negatively correlated. Positive correlation leads to a small decrease in the time-averaged radiance while negative correlation is associated with an increase. The intensity level, scale of turbulence (results for $\Lambda/H=0.4$ are not shown for brevity), and system dimension each affect the radiance. The system considered is quite simple and many assumptions have been made to facilitate calculations. Broad generalizations to other systems or conditions cannot be made. However, the example suggests that turbulence effects can be quite significant, thus warranting further research in more complex systems. The results elucidate the importance of turbulent concentration and temperature fluctuations of the radiating species on radiative transfer. The results also suggest a basis for diagnostics by measuring the fluctuating radiation quantities (i.e., irradiance or radiation flux at the wall of a system).

IV. Interaction of Radiation and Turbulence in a Combustion System

A. Physical and Mathematical Model

To illustrate the turbulence/radiation interaction we consider a two-dimensional combustion system shown in Fig. 2 which extends indefinitely in the direction perpendicular to the plane of the figure. The transport processes are steady and two-dimensional. Premixed methane air burns in a furnace and Bray's model is used for combustion. The flows are elliptic and boyancy effects due to the gravitational force are accounted for. Radiation transport from the combustion products and coupling between radiation and turbulence is considered in the analysis. Information on turbulence/radiation interactions and turbulence properties are obtained using the conserved-scalar formalism in conjunction with the standard $k-\epsilon$ turbulence model. The Favre-averaged formulation is adopted, which is conventional practice for Reynolds-averaged analysis.[18]

The solution of the problem is obtained by solving governing equations for conservation of mass, momentum, and fresh mixture temperature as well as additional modeled equations for turbulent kinetic energy k and the rate of dissipation of turbulent kinetic energy ϵ. The governing equations for these quantities can be written in the following general form:

$$\frac{\partial}{\partial x_j}(\bar{\rho}\tilde{u}_j\tilde{\phi}_j) = \frac{\partial}{\partial x_j}\left(\Gamma_\phi \frac{\partial \tilde{\phi}}{\partial x_j}\right) + S_\phi \tag{12}$$

The formulation is based on Favre mean (mass weighted) average quantities $\tilde{\phi}$, e.g.,

$$\tilde{\phi} = \overline{\rho\phi}/\bar{\rho} \tag{13}$$

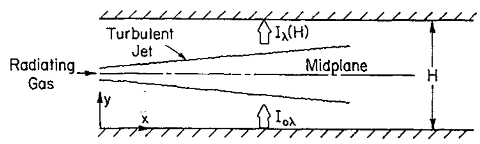

Fig. 2 Schematic diagram of combustion furnace.

where an overbar represents a conventional time average.[18] The expressions for $\tilde{\phi}_j$, Γ_ϕ, and S_ϕ are summarized in Table 3. The radiative transfer model together with the source terms for the total enthalpy $\tilde{H}$, fresh mixture temperature $\tilde{T}_1$, and the irradiance $\overline{G}_{ij}$ for the spectral groups will be discussed in the following subsection. The empirical constants appearing in the $k-\epsilon$ turbulence model are listed in Table 3.

No-slip boundary conditions were imposed at the walls for all velocity components. An impermeable boundary condition at the walls was employed for the mass fraction of fuel. At the inlet, k and ϵ were specified according to the suggestion given in the literature.[19] The temperature of the load (sink) at the bottom of the furnace is assumed to be given, and the temperature boundary condition at the remaining walls is specified by

$$-k\frac{\partial T}{\partial x_n} - q_w^r = U_\infty (T_w - T_\infty) \tag{14}$$

where U_∞ denotes the overall heat transfer coefficient between the wall surface on the inside of the furnace and the ambient. The outlet of the furnace was assumed to be adiabatic for all of the quantities except the mass flow.

B. Radiative Transfer Model

The differential and multiflux approximations of radiative transfer based on moment methods offer significant simplifications and have been recommended for use in the

Table 3 Definition of variables, diffusion coefficients, and source terms[a]

ϕ	Γ_ϕ	S_ϕ
$\tilde{u}_1$	$\mu_{\rm eff}$	$\frac{\partial}{\partial x_1}\left[p + \frac{2}{3}\left(\bar{\rho}k + \mu_{\rm eff}\frac{\partial \tilde{u}_j}{\partial x_j}\right)\right] + \frac{\partial}{\partial x_j}\left(\mu_{\rm eff}\frac{\partial \tilde{u}_j}{\partial x_1}\right)$
$\tilde{u}_2$	$\mu_{\rm eff}$	$\frac{\partial}{\partial x_2}\left[p + \frac{2}{3}\left(\bar{\rho}k + \mu_{\rm eff}\frac{\partial \tilde{u}_j}{\partial x_j}\right)\right] + \frac{\partial}{\partial x_j}\left(\mu_{\rm eff}\frac{\partial \tilde{u}_j}{\partial x_2}\right) - \bar{\rho}g$
k	$\mu_{\rm eff}/\sigma_k$	$\mu_{\rm eff}\frac{\partial \tilde{u}_i}{\partial x_j}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right) - \bar{\rho}\epsilon$ $-\frac{2}{3}\left(\bar{\rho}k + \mu_{\rm eff}\frac{\partial u_j}{\partial x_j}\right)\frac{\partial \tilde{u}_j}{\partial x_j} - \frac{\mu_{\rm eff}}{\bar{\rho}^2}\frac{\partial \bar{\rho}}{\partial x_j}\frac{\partial \bar{p}}{\partial x_j}$
ϵ	$\mu_{\rm eff}/\sigma_\epsilon$	$C_1\frac{\epsilon}{k}\left[\mu_{\rm eff}\frac{\partial \tilde{u}_i}{\partial x_j}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)\right] - C_2\bar{\rho}\frac{\epsilon^2}{k}$ $-C_1\frac{\epsilon}{k}\left[\frac{2}{3}\left(\bar{\rho}k + \mu_{\rm eff}\frac{\partial \tilde{u}_j}{\partial x_j}\right)\frac{\partial \tilde{u}_j}{\partial x_j} + \frac{\mu_{\rm eff}\partial\bar{\rho}\partial\bar{p}}{\bar{\rho}^2\partial x_j \partial x_j}\right]$
$\tilde{M}_{\rm fu}$	$\mu_{\rm eff}/\sigma_{fu}$	$-C_B\frac{\epsilon}{k}\bar{\rho}\tilde{M}_{\rm fu}(1 - \tilde{M}_{\rm fu}/M_{{\rm fu}_{\rm in}})\tilde{T}_1^2$
$\tilde{H}$	$\mu_{\rm eff}/\sigma_H$	$\bar{\kappa}_{ij}\overline{G}_{ij} - 4\overline{\kappa_{ij}w_{ij}E_b}$
$\tilde{T}_1$	$\mu_{\rm eff}/\sigma_T$	$(\tilde{T}_1/c_{p1}\tilde{T})[\kappa_{ij}(\tilde{T}_1)\overline{G}_{ij} - 4\kappa_{ij}(\tilde{T}_1)w_{ij}(\tilde{T}_1)E_b(\tilde{T}_1)]$
$\overline{G}_{ij}$ ($i=1,2$; $j=12$)	$1/\bar{\kappa}_{ij}$	$-3\bar{\kappa}_{ij}\overline{G}_{ij} + 12\overline{\kappa_{ij}w_{ij}E_b}$

C_D	C_1	C_2	C_B	σ_κ	σ_ϵ	$\sigma_{\rm fu}$	σ_H	σ_T
0.09	1.44	1.92	$1.1\times10^{-5}/{\rm K}^2$	1.0	1.30	0.9	0.9	0.9

[a] Repeated indices mean summation, except for $\overline{G}_{ij}$.

study of turbulence/radiation interaction.[8,15] This approach is considered in the following. The spectral radiative transfer calculations do not appear to be practical for a combustion system, and the gray calculations which have been used in the past do not appear to predict radiation transfer realistically.[12] Therefore, Hottel's weighted sum-of-the gray gas model[20] is generalized and used in the calculations. The details related to the spectral dependence of the absorption coefficient are given elsewhere.[21]

To simplify radiative transfer calculations in a combustion system we assume that the individual eddies are homogeneous and optically thin (i.e., the optical dimension of the radiating gas of a typical size eddy based on macroscale of turbulence is smaller than unity). We further assume that the eddies are statistically independent. This implies that there is no correlation between temperature and concentration with each eddy. Under these conditions radiation is transmitted through an eddy with little change so that the radiance at a local point is affected little by the local fluctuation of κ_λ. Hence, the time-averaged RTE can be approximated as

$$(\vec{s}\cdot\nabla)\bar{I}_\lambda = -\bar{\kappa}_\lambda \bar{I}_\lambda + \overline{\eta_\lambda} \tag{15}$$

Following a similar argument, the spectral radiant energy equation, (2) can be expressed as

$$\nabla\cdot\bar{F}_\lambda = -\bar{\kappa}_\lambda \overline{G}_\lambda + 4\pi\overline{\eta_\lambda} \tag{16}$$

Integration of the RTE over the width of the spectral group ij from A_n to B_n and then time-averaging yields the following (however, in Eqs. (17) and (19), the double indices of ij do not denote a summation):

$$(\vec{s}\cdot\nabla)\bar{I}_{ij} = -\bar{\kappa}_{ij}\bar{I}_{ij} + \overline{\kappa_{ij}w_{ij}I_{bij}} \tag{17}$$

where

$$I_{ij} = \sum_n \int_{A_n}^{B_n} I_\lambda \mathrm{d}\lambda; \quad w_{ij} = \sum_n \int_{A_n}^{B_n} I_{b\lambda}\mathrm{d}\lambda / I_b \tag{18}$$

Using the P_1 approximation for radiative transfer,[19] after some tedious mathematical manipulations we obtain the following time-averaged equation for the irradiance $\overline{G}_{ij}$,

$$\frac{\partial}{\partial x_k}\left(\frac{1}{3\bar{\kappa}_{ij}}\frac{\partial \overline{G}_{ij}}{\partial x_k}\right) = \bar{\kappa}_{ij}\overline{G}_{ij} - 4\overline{\kappa_{ij}w_{ij}E_b} \tag{19}$$

The radiant heat flux at the wall q_{wij} is related to the irradiance $\overline{G}_{ij}$ as

$$q_{wij} = \frac{1}{3\bar{\kappa}_{ij}}\frac{\partial \overline{G}_{ij}}{\partial x_n} = \left[-\frac{2\epsilon_w}{2-\epsilon_w}\overline{w_{ij}E_b} + \frac{\epsilon_w}{2(2-\epsilon_w)}\overline{G}_{ij}\right]_w \tag{20}$$

where x_n is a coordinate normal to the surface and ϵ_w is the wall emissivity.

We note that the P_1 approximation as well as the higher order approximations[22] are of the anisotropic diffusion type and can be written in the form

$$\frac{\partial}{\partial x_j}\left(\Gamma_\phi \frac{\partial\bar{\phi}}{\partial x_j}\right) + S_\phi = 0 \tag{21}$$

The time-averaged irradiances of the spectral groups, i.e., $\overline{G}_{11}$, $\overline{G}_{12}$, $\overline{G}_{21}$, and $\overline{G}_{22}$, along with the sources, are given in Table 3. The boundary conditions for Eq. (19) are specified by Eq. (20). In the table κ_{ij} and w_{ij} are the absorption coefficient and weighting factor respectively, for group ij. The time-mean quantities were obained as

$$\bar{f} = \tau f_1 + (1-\tau) f_2 \tag{22}$$

where f_1 denotes the quantity for the fresh mixture, and f_2 is the corresponding quantity for the product. The time τ the fresh mixture remains unreacted is related to the Favre mean defined as

$$\tau = (\tilde{M}_{\mathrm{fu}}/M_{\mathrm{fu,in}})(\bar{\rho}/\rho_1) \tag{23}$$

C. Method of Solution

The model equations, (14) and (19), together with some auxiliary equations and boundary conditions, were solved iteratively using a version of SIMPLER algorithm for integating numerically a system of elliptic partial differential equations. The details are available elsewhere.[21] It is sufficient to mention that, in order to reduce the number of grid points and improve the resolution, wall functions were used for velocity components, total enthalpy, fuel mass fraction, turbulent kinetic energy, and rate of dissipation of turbulent kinetic energy.

D. Numerical Results and Discussion

1. Model Combustion System and Parameters

The parameters selected for the performance calculations of a model furnace are as follows:

$H = 1$ m,	$L = 5$ m
$T_s = 1500$ K	$T_\infty = 300$ K
$\epsilon_s = 0.8$	$\epsilon_w = 0.6$
$Q_{\mathrm{in}} = 2.5$ MW	$T_{\mathrm{in}} = 1000$ K
$U_\infty = 5$ W/m^{2}K,	$u_{\mathrm{in}} = 10.9$ m/s

The combustion system was assumed to burn a mixture of methane and 10% excess air to heat a sink at the bottom of the furnace (see Fig. 2). The incoming fuel-air mixture is preheated to a temperture of 1000 K, and the heat input to the furnace was assumed to be 2.5 MW/m of furnace width. The model furnace design and operating parameters will be changed from the base conditions indicated above, and these perturbations will be duly noted as the results are discussed.

2. Discussion of Results for Base Simulation

The flowfield and turbulence quantities for the numerical simulations with turbulence/radiation interaction are given in Fig. 3. Comparison of corresponding panels in the figures shows that there is relatively little difference between the results for the flow parameters for the two simulations, one with and another without turbulence/radiation interaction. This is because only the temperature affects the flowfield by changing the local density so that the small difference in the predicted temperature between the two models has little influence on the flowfield. The gas temperature is generally higher than that calculated when turbulence/radiation interaction is accounted. This is because the emission of radiation by a gas with a nonfluctuating temperature is lower than in the fluctuating case, $[\overline{\kappa I_b}/\bar{\kappa}I_b(\bar{T})] > 1$. The results show that the emission near the flame is much stronger than the one calculated at the mean scalar values. The fluctuating gas emits more energy than the nonfluctuating and hence the gas has a lower temperature than the latter. Transport of energy diffusion is much smaller than by radiation.

The streamlines (Fig. 3a) indicate two dominant recirculation cells while the incoming flow gradually expands as it proceeds along the furnace. Since the density decreases rapidly across the flame as the fuel burns, the bulk velocity also increases (Fig. 3b). The effect of buoyancy is found to be negligible since the force is much smaller than the inertia force. The existence of recirculation in a combustion system of this type is confirmed by many experiments, and the flowfield is similar to the flowfield over a backward facing step. Recirculation near the exit is indiscernible, and this finding is consistent with experimental observations.[23] This is because of very high turbulent viscosity near the exit. The effective viscosity is given by $\mu_{eff} = \mu + C_D \rho (k^2/\epsilon)$. The turbulent viscosity near the center of the furnace is about three orders of magnitude larger than the molecular viscosity.

The production of turbulent kinetic energy is intense near the flame where large velocity gradients exist. The turbulent kinetic energy is smaller along the centerline of the furnace than at the shear layer due to smaller production of the turbulence along the centerline. The dissipation of turbulent kinetic energy ϵ is a very complicated function; however, it can be considered to be large where turbulent kinetic energy is large. The results obtained allow us to conclude that the effect of turbulence/radiation interaction on the flow properties in the combustion system considered can be neglected. The mean flow quantities are not sensitive to the turbulence/radiation interaction. The results obtained are very close to those in the absence of interaction, and therefore are not given in this paper but can be found elsewhere.[21]

The results for the local convective, radiative, and total (convective plus radiative) fluxes at the sink surface clearly show (Fig. 4a) that convective heat flux is very small (less than 1%) compared to the radiative flux and can therefore be neglected. This finding is in agreement with the predictions based on phenomenological industrial furnace performance models and measurements.[24] A comparison of the local fluxes predicted to the sink (bottom of furnace) for the simulations with and without turbulence/radiation interaction is given in Fig. 4b. In spite of the fact that the emission of radiation for the case when turbulence/radiation interaction is accounted for is much greater near the flame, the total (convective plus radiative) heat flux to the sink is only 8% higher at the inlet. For $x/L > 0.4$ the local fluxes are practically identical. The difference between the average fluxes for the two simulations is only about 1%. The reason for this is that the strong fluctuating emission is primarily confined to the small volume of the flame and the fact that the cool fresh mixture located close to the burned gas absorbs the radiation emitted by the products.

3. Summary of Parametric Calculations

To determine the effect of turbulence/radiation interaction when the flame is long, a set of calculations with and without interaction was made for the fresh mixture inlet temperature of 300 K. The energy input to the system was 2.5 MW/m. The density of fresh mixture is much greater than for the preheated case. This results in smaller inlet velocity and the recirculation cell is smaller. The flow pattern has the characteristics of natural convection as can be found from the streamlines for the calculation with turbulence/radiation interaction as depicted in Fig. 5a. The turbulence intensity is about 180%. This high intensity is the result of the significant density change of the gas due to combustion. The flame extends over 2/3 of the furnace length. As can be found from Fig. 5b, the mean temperature of the gas increases rather slowly owing to the long flame and slow combustion. The maximum mean temperature is 1853 K and it occurs near the roof in the middle of furnace. The fluxtuations of temperature vary from 200 to 700 K and are generally high over the entire combustion space. Only 15% of the heat input to the furnace is transferred to the sink.

The calculation without turbulence/radiation interaction with the same flame shape as the one with turbulence/radiation interaction shows little change in the flow pattern. However, the heat flux distribution at the sink is significantly different as can be seen from the comparisons given in Fig. 6. Upstream, for $T_{in} = 300$ K the heat flux is even negative for the simulation without turbulence/radiation interaction, whereas the calculation with the turbulence/radiation interaction predicts positive radiation heat flux from the combustion space to the sink. The net heat transfer to the sink is predicted

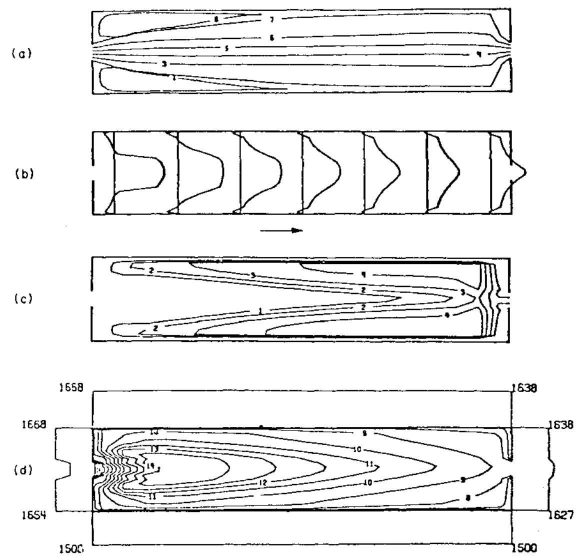

Fig. 3 Flowfield variables for the simulation with turbulence/radiation interaction: a) streamline contours: 1,2,3,4,5,6,7,8 correspond to −5,0,20,40,60,80,100,105 (in percent of mass flow rate at inlet); b) sectional view of u-velocity distributions; c) effective viscosity distributions; d) temperature distributions in the furnace and at the boundaries (1,2,...14 correspond to 1100 K, 1200 K,...2400 K).

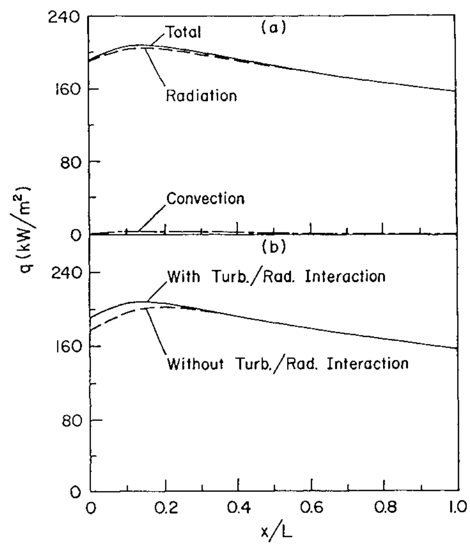

Fig. 4 Lock heat flux distributions along the sink: a) comparison of local convective and radiative fluxes for a simulation with turbulence/radiation interaction; b) comparison of the total (convective plus radiative) fluxes for simulations with and without turbulence/radiation interaction.

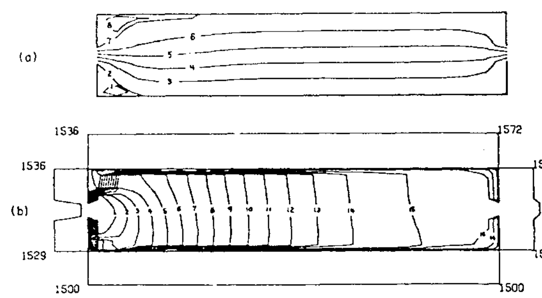

Fig. 5 Flowfield variables for fresh mixture temperature of 300 K with turbulence radiation interaction: a) streamline contours 1,2,3,4,5,6,7,8 correspond to −5,0,20,40,60,80,100,105 (in percent of incoming mass flow rate); b) temperature distributions 1,2,...15 correspond to 400, 500, ...1800 K.

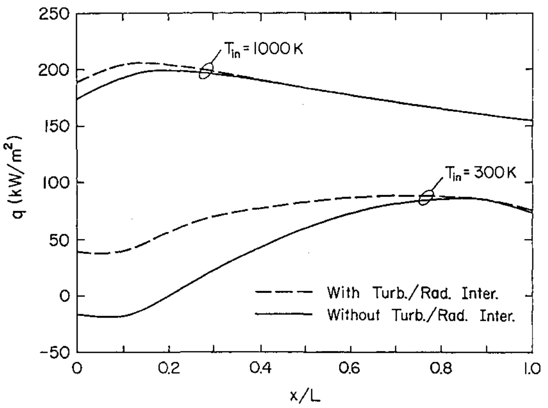

Fig. 6 Comparison of the total heat flux distributions along the sink with and without turbulence/radiation interaction for a fresh mixture temperature of 300 K.

to be only 9.5% of the total heat input for the noninteracting case. Because of the relatively low-heat fluxes for these two computations, the fraction of convective heat transfer rate to the total heat transfer rate is higher—2.2% for the interacting case and 8.8% for the noninteracting one.

The results clearly show that the effect of turbulence/radiation interaction is significant when the flame is large in size compared to the volume of the furnace. The gas emits more radiant energy than that calculated on the basis of the mean temperature due to turbulent fluctuations.

V. Conclusions

Based on the results of sample calculations, we can make the following conclusions:

1) The irradiance on a wall of a parallel-plate channel confining a two-dimensional turbulent jet depends on the turbulent conditions. Of profound importance is whether the concentration of the radiating species and emission fluctuations are positively or negatively correlated. Positive correlation is found to lead to small decreases in time-averaged irradiances while negative correlation is associated with increases. The intensity levels, scale of turbulence, and system dimensions each affect the magnitude of the increases or decreases in irradiance in comparison to that based on the mean properties.

2) Within the stochastic method, radiation transfer can be calculated by assuming as a first approximation that the concentration of the radiating species and the local radiance are statistically independent. The results of calculations show that when the flame occupies a relatively small volume fraction of the combustion system, the effect of turbulence/radiation interaction on the scalar properties and the total heat transfer to the sink is negligible. Whereas, then the flame occupies a relatively large volume fraction of the combustion system, the interaction is quite significant.

3) Differential formulations of radiative transfer offer significant advantages for practical calculations involving turbulence/radiation interactions; however, their continued development and improvement is needed. The conclusions reached should be checked using more detailed turbulence/radiation interaction models, and the results need to be verified experimentally to provide confidence in the predictions.

4) There is a need for extension of the assumption that the absorption coefficient (radiating species concentrations) and the radiation field are statistically independent. Recent advances in numerical simulations of time-dependent, chemically reacting turbulent flows should be exploited to study turbulence/radiation interaction to provide a fundamental understanding of this process.

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